

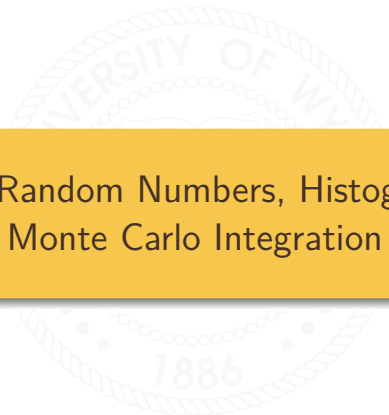
MATH 3341: Introduction to Scientific Computing Lab

Libao Jin

University of Wyoming

December 4, 2019





Lab 13: Random Numbers, Histogram and Monte Carlo Integration



Random Numbers and Histogram



rand: Uniformly distributed pseudorandom numbers.

- $R = \text{rand}(N)$ returns an N -by- N matrix containing pseudorandom values drawn from the standard uniform distribution on the open interval $(0,1)$.
- $\text{rand}(M,N)$ or $\text{rand}([M,N])$ returns an M -by- N matrix.
- $\text{rand}(M,N,P,\dots)$ or $\text{rand}([M,N,P,\dots])$ returns an M -by- N -by- P -by- \dots array.
- rand returns a scalar.
- $\text{rand}(\text{SIZE}(A))$ returns an array the same size as A .



randn: Normally distributed pseudorandom numbers.

- $R = \text{randn}(N)$ returns an N -by- N matrix containing pseudorandom values drawn from the standard normal distribution.
- $\text{randn}(M,N)$ or $\text{randn}([M,N])$ returns an M -by- N matrix.
- $\text{randn}(M,N,P,\dots)$ or $\text{randn}([M,N,P,\dots])$ returns an M -by- N -by- P -by-... array. randn returns a scalar.
- $\text{randn}(\text{SIZE}(A))$ returns an array the same size as A .



`randi`: Pseudorandom integers from a uniform discrete distribution.

- `R = randi(IMAX,N)` returns an N -by- N matrix containing pseudorandom integer values drawn from the discrete uniform distribution on $1:IMAX$.
- `randi(IMAX,M,N)` or `randi(IMAX,[M,N])` returns an M -by- N matrix.
- `randi(IMAX,M,N,P,...)` or `randi(IMAX,[M,N,P,...])` returns an M -by- N -by- P -by-... array.
- `randi(IMAX)` returns a scalar.
- `randi(IMAX,SIZE(A))` returns an array the same size as A .



histogram: Plots a histogram.

- `histogram(X)` plots a histogram of X . `histogram` determines the bin edges using an automatic binning algorithm that returns uniform bins of a width that is chosen to cover the range of values in X and reveal the shape of the underlying distribution.
- `histogram(X,M)`, where M is a scalar, uses M bins.
- `histogram(X,EDGES)`, where $EDGES$ is a vector, specifies the edges of the bins.



The background of the slide features a large, faint watermark of the University of Wyoming seal. The seal is circular with a rope-like border. Inside the border, the words "UNIVERSITY OF WYOMING" are at the top, "EQUALITY" is in the center, and "1886" is at the bottom. In the middle of the seal is an open book.

Monte Carlo Integration



1-D Monte Carlo Integration

$$\begin{aligned}\int_a^b f(x) dx &= \int_a^b \frac{f(x)}{p(x)} p(x) dx \\ &= E[f(X)/p(X)] \\ &\approx \frac{1}{N} \sum_{i=1}^N \frac{f(x_i)}{p(x_i)} \\ &= \frac{1}{N} \sum_{i=1}^N \frac{f(x_i)}{1/(b-a)} \\ &= \frac{b-a}{N} \sum_{i=1}^N f(x_i),\end{aligned}$$

where x_1, x_2, \dots, x_N are uniformly distributed on $[a, b]$, hence $p(x_i) = \frac{1}{b-a}, i = 1, 2, \dots, N$.



2-D Monte Carlo Integration

$$\begin{aligned}\int_a^b \int_c^d f(x, y) dy dx &= \int_a^b \int_c^d \frac{f(x, y)}{p(x, y)} p(x, y) dy dx \\&= \int_a^b \int_c^d \frac{f(x, y)}{p_X(x)p_Y(y)} p_X(x)p_Y(y) dy dx \\&= \int_a^b \frac{1}{p_X(x)} \int_c^d \frac{f(x, y)}{p_Y(y)} p_Y(y) dy p_X(x) dx \\&= \int_a^b \frac{1}{p_X(x)} E[f(x, Y)/p_Y(Y)] p_X(x) dx \\&\approx \int_a^b \frac{1}{p_X(x)} \frac{1}{N} \sum_{j=1}^N \frac{f(x, y_j)}{p_Y(y_j)} p_X(x) dx \\&\approx \frac{1}{M} \sum_{i=1}^M \frac{1}{p_X(x_i)} \frac{1}{N} \sum_{j=1}^N \frac{f(x_i, y_j)}{p_Y(y_j)}\end{aligned}$$



2-D Monte Carlo Integration

$$\begin{aligned}\int_a^b \int_c^d f(x, y) dy dx &\approx \frac{1}{M} \sum_{i=1}^M \frac{1}{p_X(x_i)} \frac{1}{N} \sum_{j=1}^N \frac{f(x_i, y_j)}{p_Y(y_j)} \\ &= \frac{1}{MN} \sum_{i=1}^M \frac{1}{1/(b-a)} \sum_{j=1}^N \frac{f(x_i, y_j)}{1/(d-c)} \\ &= \frac{(b-a)(d-c)}{MN} \sum_{i=1}^M \sum_{j=1}^N f(x_i, y_j),\end{aligned}$$

where X and Y are independent and identically uniformly distributed, hence $p(x, y) = p_X(x)p_Y(y)$, and $p_X(x) = \frac{1}{b-a}$, $p_Y(y) = \frac{1}{d-c}$.

